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U(6, 6) symmetry and the generalized Veneziano model for $K^-p \to K^{*-}\pi^+n$

D BARKAI† and K J M MORIARTY‡

† Physics Department, Imperial College of Science and Technology, Prince Consort Rd, London SW7, UK
‡ Department of Mathematics, Royal Holloway College, Englefield Green, Surrey, UK

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Abstract. U(6, 6) symmetric kinematic factors are combined with the generalized Veneziano model. The model is applied to $K^-p \to K^{*-}\pi^+n$. Some arbitrariness of parameters is removed and comparisons with the data show the results are at least as good as in previous models.

1. Introduction

Some time ago a model for the study of three-particle production data was proposed by Petersson and Törnqvist (1969). In their model the sum over the 12 five-point Veneziano terms which describes the amplitude is multiplied by a kinematic factor K so that

$$A = K\sum_{1}^{12} B_5$$

where K, for the processes considered (Petersson and Törnqvist 1969), was taken to be

$$K = \epsilon_{\alpha\beta\gamma\delta}p_1^\alpha p_2^\beta p_3^\gamma p_4^\delta$$

so that it gives an overall abnormal parity and where p_i are any four of the external particle momenta. Despite the considerable success of this model no account was taken of the spin or isospin of the external particles. Attempts to take them into account were made by constructing the supermultiplet Veneziano model (Mandelstam 1969, Delbourgo and Rotelli 1969, Bardacki and Halpern 1969). Application of the model (see also Adjei *et al* 1971a, 1972) amounts to giving a separate kinematic factor to each of the five-point Veneziano terms, or, equivalently, giving each B_5 term a different weight in phase-space. We take now

$$A = \sum_{1}^{12} K_i B_5^i$$

where each K_i is evaluated by employing the U(6,6) multispinor formalism (Salam *et al* 1965, Sakita and Wali 1965, Delbourgo *et al* 1965) and applying it to the corresponding 'legal' Harari–Rosner duality diagram (Harari 1969, Rosner 1969). The formalism will be briefly discussed in the next section.

It is amusing to note that in all previous applications, and presumably in all similar processes, the leading term, when taking the corresponding double-Regge-limit, is of the type of the kinematic factor K considered by Petersson and Törnqvist.

We apply the 'U(6,6)+Veneziano' model to the process $K^-p \to K^{*-}\pi^+n$. This process was considered before, using the Petersson–Törnqvist kinetic factor (Collins *et al* 1970, Adjei *et al* 1971b). Firstly we are trying to see if there is a better fit to the experimental data than with the old model; and secondly, to see if there are any lessons to be learnt by applying the model for the first time to a process with an external spin one particle, in addition to the two spin half particles. It is found that the mass and momentum-transfer distributions fit the data reasonably well, but the algebraic complexity in evaluating the amplitude increases considerably. Furthermore, there is no one B_5 term which dominates in all regions, as was the case in previous processes considered. This is found by looking at that mass distribution which gets a resonance contribution from both the terms of the amplitude we take for $K^-p \to K^{*-}\pi^+n$ (see § 4).

2. Formalism. Construction of the kinematic factors

To evaluate the kinematic factors K_i in the amplitude

$$A = \sum K_i B_5^i$$

we employ the U(6,6) multispinor formalism (Salam *et al* 1965, Sakita and Wali 1965, Delbourgo *et al* 1965). All mesons are $q\bar{q}$ combinations of spin half, SU(3) triplet quarks. They belong to the $(6,\bar{6})^-$ supermultiplet and are described by the wavefunction

$$\Phi_A^B(k) \equiv \Phi_{\alpha a}^{\beta b}(k) = \left(\frac{1}{2\mu}\right)((\not{k}+\mu)(\gamma_\mu \phi_{\mu a}^{\ b-}\gamma_5 \phi_{5a}^{\ b}))_\alpha^\beta$$

where α, β are Dirac spinor indices, and a, b are SU(3) indices. Similarly, a baryon is described by a completely symmetric combination of qqq and its wavefunction is given by

$$\begin{aligned}\psi_{ABC}(p) &\equiv \psi_{\alpha a,\beta b,\gamma c}(p)\\ &= \frac{1}{2m}\sqrt{\tfrac{2}{3}}((\not{p}+m)\gamma_\mu C)_{\alpha\beta}D_{\mu\gamma(abc)} + \frac{1}{2m}\sqrt{\tfrac{1}{6}}\{((\not{p}+m)\gamma_5 C)_{\alpha\beta}\epsilon_{abs}N_{\gamma c}{}^s\\ &\quad + (\text{cyclic permutation in } \alpha a, \beta b, \gamma c)\}\end{aligned}$$

(for notation see Salam *et al* 1965).

The kinetic term K_i is the U(6,6) amplitude that we get by drawing the duality diagram corresponding to the ith ordering of the external particles, which are represented by quarks. This corresponds to a given way of contracting the quark indices A, B etc which we deduce by following the flow of the quark lines (see figure 2). From Φ_A^B and ψ_{ABC} we will take only the SU(3) term in which the given external particle in the process under consideration is included. The U(6,6) symmetry is used only to construct a kinematic factor. All the dynamics, the resonance structure, we assume to be given by the B_5 function. A more detailed calculation of the kinematic factors for the process $K^-p \to K^{*-}\pi^+n$ can be found in the Appendix.

3. Application to $K^-p \rightarrow K^{*-}\pi^+n$

We take for the amplitude the contribution from the two diagrams shown in figure 1. This is dictated by the absence of exotic mesons (no diagram where K^- and π^+ are adjacent) and the fact that no trajectory, which couple to the K^-n or $K^{*-}p$ subsystems, was found. Following the reasoning of Collins *et al* (1970) and Adjei *et al* (1971b) we assume all particles to lie on linear Regge trajectories, all having the same slope.

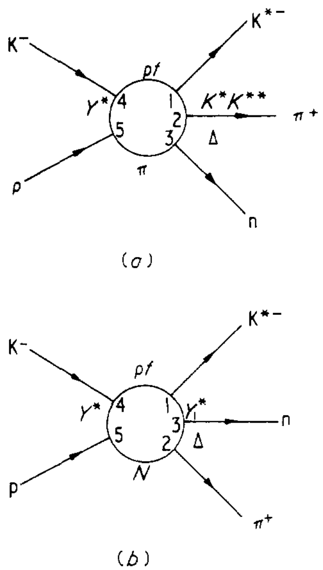

Figure 1. The two permutations contributing to the amplitude.

The amplitude can be written then as

$$A = \sum_{i=a,b} K_i B_5^i(X_{12}, X_{23}, X_{34}, X_{45}, X_{51})$$

with $X_{ij} = J - \alpha_{ij}$. α_{ij} is the trajectory coupled to the external particles i and j and J is the spin of the lowest resonance on this trajectory. The only exception is the shift by $\frac{1}{2}$ of the argument of the N_α in order to get correct asymptotic behaviour. The choice of trajectories has been discussed in Collins *et al* (1970). We list them here again with only slight changes corresponding to changes of some experimental values for masses and widths of resonances

$$\alpha_{K^*}(s_{K^*\pi}) = 0{\cdot}18 + 0{\cdot}95 + i0{\cdot}10(s_{K^*\pi} - s_0) \qquad s_0 = (m_{K^*} + M_\pi)^2$$

$$\alpha_\rho(t_{K^*K^-}) = 0{\cdot}48 + 0{\cdot}9t_{K^*K^-}$$

$$\alpha_\Sigma(s_{K^-p}) = -0{\cdot}22 + 0{\cdot}9s_{K^-p} + i0{\cdot}13(s_{K^-p} - s_0) \qquad s_0 = (m_p + m_K)^2$$

$$\alpha_N(t_{p\pi}) = -0{\cdot}3 + 0{\cdot}9t_{p\pi}$$

$$\alpha_\pi(t_{pn}) = -0{\cdot}0175 + 0{\cdot}9t_{pn}$$

$$\alpha_\Delta(s_{n\pi}) = 0{\cdot}12 + 0{\cdot}9s_{n\pi} + i0{\cdot}36(s_{n\pi} - s_0) \qquad s_0 = (m_n + m_\pi)^2$$

$$\alpha_\Sigma(s_{K^*n}) = -0{\cdot}22 + 0{\cdot}9s_{K^*n} + i0{\cdot}3(s_{K^*n} - s_0) \qquad s_0 = (m_{K^*} + m_n)^2.$$

An imaginary part has been added to the trajectories above threshold to account for the width of the resonances.

The complexity that arises from having an external spin one particle becomes apparent when one evaluates the kinematic factors according to the formalism described in the previous section. In figure 2 the two duality diagrams corresponding to the diagrams of figure 1 are shown. The flow of the quark lines is indicated. Following this flow and considering all mesons as ingoing we arrive at

$$K_a = \bar{\psi}^{ABC}(\mathrm{n})\phi^D_C(-\pi^+)\phi^E_D(-\mathrm{K}^{*-})\phi^F_E(\mathrm{K}^-)\psi_{FBA}(\mathrm{p})$$

$$K_b = \bar{\psi}^{ABC}(\mathrm{n})\phi^F_A(-\pi^+)\phi^D_B(-\mathrm{K}^{*-})\phi^E_D(\mathrm{K}^-)\psi_{ECF}(\mathrm{p})$$

where the arguments stand for the momenta of the particles indicated. The relevant term for the π^- and K wavefunction is the SU(3) ϕ_5 term, that for K* is the SU(3) ϕ_μ term and for the spinors p and n it would be enough to consider the three terms in ψ_{ABC} with the SU(3) matrix N_α.

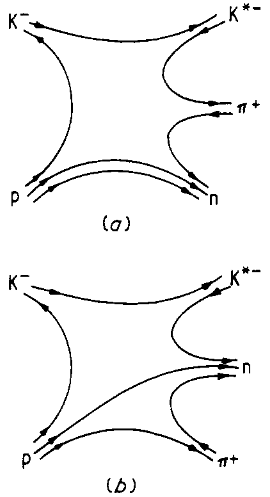

Figure 2. The duality diagrams corresponding to the permutations of figure 1.

To evaluate K_a and K_b we separate the SU(3) and the U(2,2) (Dirac) traces. There are nine terms in each K_a and K_b. We take each such term, evaluate its SU(3) trace, multiply it by its U(2,2) trace and then add them up to give the K_i. Having the polarization vector of K^{*-} in addition to the four independent external momenta increases the complexity of the resulting expressions considerably. The explicit expressions for K_a and K_b with the details of their evaluation are given in the Appendix. For U(6,6) mass splitting we take for μ, the meson mass, the average over the 0^- octet and the 1^- nonet, and for m, the baryon mass, we take the average of the $\frac{1}{2}^+$ octet.

4. Comparison with experiment and concluding remarks

Having the amplitude, the theoretical distributions are calculated using the program FOWL (F James *CERN Program Library* no. W505, unpublished) with Hopkinson's program (J F L Hopkinson 1969 *Daresbury Rep.* no. DNPL/P21, unpublished) for B_5.

The predictions are compared with data at 6 and 10 GeV/*c*, and are shown in figures 3–6. The data were taken from the results of the Birmingham–Glasgow–London (IC)–Munich–Oxford collaboration's exposure at 6·0 GeV/*c* and the Aachen–Berlin–CERN–

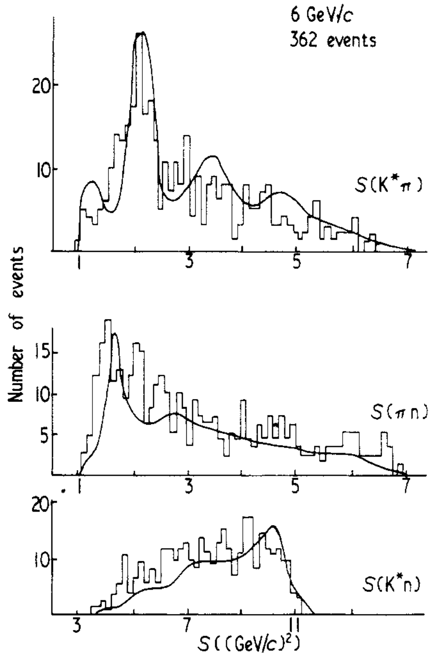

Figure 3. Mass squared distribution at 6·0 GeV/*c* for $K^-p \to K^{*-}\pi^+n$.

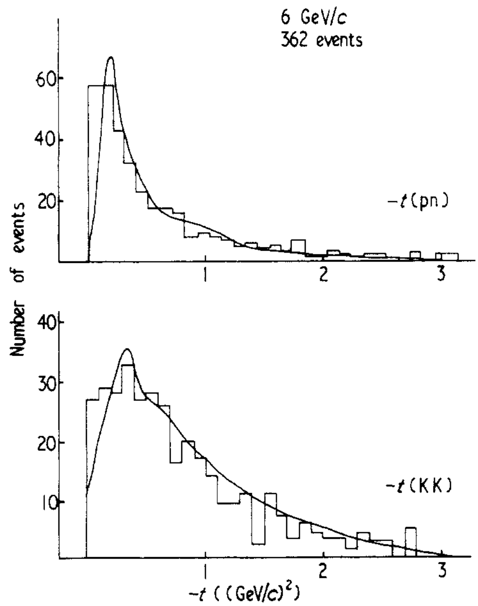

Figure 4. Momentum transfer distributions at 6·0 GeV/*c* for $K^-p \to K^{*-}\pi^+n$.

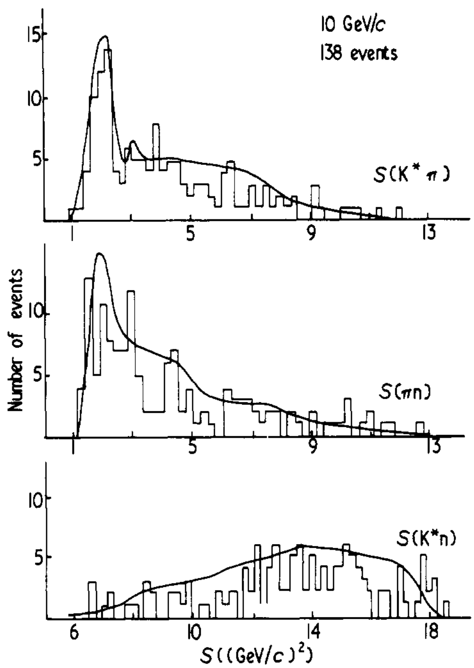

Figure 5. Mass squared distributions at 10·0 GeV/c for $K^-p \to K^{*-}\pi^+n$.

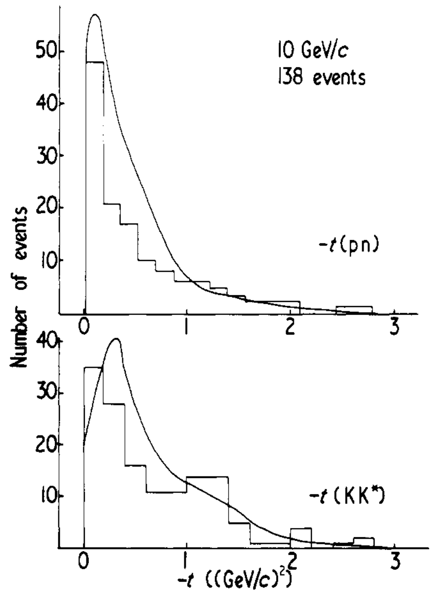

Figure 6. Momentum transfer distributions at 10·0 GeV/c for $K^-p \to K^{*-}\pi^+n$.

London(IC)–Vienna collaboration's exposure at 10 GeV/c. The experimental histograms were taken from earlier treatment of this process (see Hunt and Schonfelder 1969). The same bin width as in the experiment was used. For our amplitude (see Appendix) a long computing time was needed, so that histograms had to be weighted and summed up (using different regions of the random numbers generated by FOWL, of course) in order to get the general form for a curve that can then be smoothed by hand. We have fixed the normalization parameter using the data at 6 GeV/c.

In general we consider our results to be an improvement over those obtained by Collins *et al* 1970 and by Hunt and Schonfelder (1969) and slightly better than those of Adjei *et al* (1971b). This was obtained even though the number of free parameters has been reduced by imposing the U(6,6) symmetry upon the kinematic factors. The size of the peaks for $s(K^*\pi)$ and $s(\pi n)$ fits better than in Adjei *et al* (1971b) (which has the best fits of all the references quoted), although the peak of $s(\pi n)$ at 6 GeV/c seems to be shifted a bit. $s(K^*n)$ at 6 GeV/c shows more structure but seems to be in poorer agreement with the data than in Adjei *et al* (1971b). The momentum transfer distributions show π exchange dominance, which results in the forward peak. The status of the data for this process does not enable us to make more conclusive statements.

The resonance structure is built in by the B_5 Veneziano amplitude and we want to test here how it is modified by our kinematic factors. Quantitatively, it has reduced the peaks and enhanced the background to fit the data better. Also, by changing the undefined phase factor of the kinematic factor (being a U(6,6) amplitude) we have found that the peak of $s(\pi n)$, which get contribution from both terms of the amplitude, has been eliminated when multiplying K_b by $e^{i\pi/2}$. It shows that K_a and K_b are of the same order of magnitude, at least when $s(\pi n) \simeq 1{\cdot}5$–2 GeV.

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Appendix

Here we evaluate the kinematic factor K_a in detail. Writing explicitly all the terms in K_a, which is given in § 3, we have

$$K_a = \frac{1}{2m\sqrt{6}}\{\epsilon^{abs}\bar{N}^{\gamma c}_{2s}(C^{-1}\gamma_5(\not{n}+m))^{\alpha\beta} + \epsilon^{bcs}\bar{N}^{\alpha a}_{2s}(C^{-1}\gamma_5(\not{n}+m))^{\beta\gamma}$$

$$+ \epsilon^{cas}\bar{N}^{\beta b}_{2s}(C^{-1}\gamma_5(\not{n}+m))^{\gamma\alpha}\}\frac{(-1)}{2\mu}((-\not{\pi}^{+}+\mu)\gamma_5)^{\delta}_{\gamma}(\phi_5(\pi^{+}))^{d}_{c}$$

$$\times\frac{1}{2\mu}((-\not{K}^{*-}+\mu)\gamma_\mu)^{\epsilon}_{\delta}(\phi_\mu(K^{*-}))\frac{e}{d}\,\frac{(-1)}{2\mu}((\not{K}^{-}+\mu)\gamma_5)^{\phi}_{\epsilon}(\phi_5(K^{-}))^{f}_{e}$$

$$\times\frac{1}{2m\sqrt{6}}\{((\not{p}+m)\gamma_5C)_{\phi\beta}\epsilon_{fbr}N'_{1\alpha a} + ((\not{p}+m)\gamma_5C)_{\beta\alpha}\epsilon_{bar}N'_{1pf}$$

$$+((\not{p}+m)\gamma_5C)_{\alpha\phi}\epsilon_{afr}N'_{1\beta b}\}$$

where N_2 (N_1) is the 'SU(3)–U(2,2)' spinor corresponding to $n(p)$, and all the mesons are considering as ingoing.

By suppressing the U(2,2) indices, and then the SU(3) indices we define the following expressions:

$$K_a^{SU(3)} = (\epsilon^{abs}\bar{N}^c_{2s} + \epsilon^{bcs}\bar{N}^a_{2s} + \epsilon^{cas}\bar{N}^b_{2s})(\tau_{\pi^+})^d_c(\tau_{K^{*-}})^e_d(\tau_{K^-})^f_e$$
$$\times (\epsilon_{fbr}N^r_{1a} + \epsilon_{bar}N^r_{1f} + \epsilon_{afr}N^r_{1b})$$

where N and τ are the SU(3) matrices for the $\frac{1}{2}^{\pm}$ spin baryon octet and 0^- meson octet, respectively, and

$$K_a^{U(2,2)} = \{\bar{N}^\gamma(n)(C^{-1}\gamma_5(\not{n}+m))^{\alpha\beta} + \bar{N}^\alpha(n)(C^{-1}\gamma_5(\not{n}+m))^{\beta\alpha} + \bar{N}^\beta(n)(C^{-1}\gamma_5(\not{n}+m))^{\gamma\alpha}\}$$
$$\times ((-\not{\pi}^+ + \mu)\gamma_5(-\not{K}^{*-} + \mu)\gamma_\mu\epsilon_\mu(\lambda_{K^{*-}})(\not{K}^- + \mu)\gamma_5)^\phi_\gamma$$
$$\times \{((\not{p}+m)\gamma_5 C)_{\phi\beta}N_\alpha(p) + ((\not{p}+m)\gamma_5 C)_{\beta\alpha}N_\phi(p) + ((\not{p}+m)\gamma_5 C)_{\alpha\phi}N_\beta(p)\}$$

where $\epsilon_\mu(\lambda_{K^{*-}})$ is the polarization vector of K^{*-} with helicity λ. Using these expressions K_a can be written as

$$K_a = \frac{1}{192\mu^3 m^2} K_a^{SU(3)} \cdot K_a^{U(2,2)}$$

and the 'scalar product' means that we take each of the nine terms in $K_a^{SU(3)}$ and multiply it by the appropriate term in $K_a^{U(2,2)}$ and then add these nine products to get K_a.

Finding the SU(3) traces is trivial, and two of them vanish. When inspecting the remaining terms of K_a we find that only two of them are different. The γ_5 and C (the particle-conjugation matrix) are easily commuted through.

Writing q_1, q_2, K for the momenta of π^+, K^{*-}, K^- respectively we find the two traces to be evaluated to be

$$\langle \bar{N}(n)((-\not{q}_1 + \mu)(\not{q}_2 + \mu)\not{\epsilon}(-\not{K} + \mu)(\not{p} + m)(\not{n} + m))N(p)\rangle$$
$$= 2(p, n + m^2)\langle \bar{N}(n)((-\not{q}_1 + \mu)(\not{q}_2 + \mu)\not{\epsilon}(-\not{K} + \mu))N(p)\rangle$$

and

$$\langle \bar{N}(n)((\not{p} + m)(-\not{K} + \mu)\not{\epsilon}(\not{q}_2 + \mu)(-\not{q}_1 + \mu)(\not{n} + m))N(p)\rangle.$$

From now on the calculation is straightforward but tedious. Using the standard techniques for evaluation of traces of γ matrices we finally get

$$\begin{aligned}
m^2\mu^3 K_a = N_0\{&4(p{\cdot}n + m^2)(13\mu^2(K{\cdot}\epsilon + q_1{\cdot}\epsilon) + 44q_2{\cdot}q_1 K{\cdot}\epsilon + 8q_2{\cdot}Kq_1{\cdot}\epsilon - 44\epsilon{\cdot}q_1 K{\cdot}q_2 \\
&- 8\epsilon{\cdot}Kq_2{\cdot}q_1) + 8m\mu(K{\cdot}q_1 - K{\cdot}q_2 + q_2{\cdot}q_1)\epsilon{\cdot}(p+n) + 8m\mu(p+n){\cdot}((q_2 - q_1)K{\cdot}\epsilon \\
&+ (K + q_2)\epsilon{\cdot}q_1) + 8K{\cdot}\epsilon(p{\cdot}q_1 n{\cdot}q_2 - p{\cdot}q_2 n{\cdot}q_1) + 8\epsilon{\cdot}q_1(p{\cdot}q_2 n{\cdot}K - p{\cdot}Kn{\cdot}q_2) \\
&+ 8K{\cdot}q_2(p{\cdot}\epsilon n{\cdot}q_1 - p{\cdot}q_1 n{\cdot}\epsilon) + 8K{\cdot}q_1(p{\cdot}q_2 n{\cdot}\epsilon - p{\cdot}\epsilon n{\cdot}q_2) \\
&+ 8q_2{\cdot}q_1(p{\cdot}\epsilon n{\cdot}K - p{\cdot}Kn{\cdot}\epsilon) + 16m\mu\epsilon{\cdot}q_1(n{\cdot}K + m\mu)\}\langle \bar{N}_2 N_1\rangle \\
&- 12(p{\cdot}n + m^2)K_\mu\epsilon_\nu q_{2\rho}q_{1\lambda}\epsilon_{\mu\nu\rho\lambda}\langle \bar{N}_2\gamma_5 N_1\rangle + \{(8\mu(p{\cdot}Kn{\cdot}(q_1 - q_2) \\
&- p{\cdot}q_2 n{\cdot}(q_1 + K) + p{\cdot}q_1 n{\cdot}(q_2 + K)) + 20(p{\cdot}n + m^2)\mu^3 \\
&+ (p{\cdot}n + m^2)\mu(20K{\cdot}q_2 - 84K{\cdot}q_1 - 20q_2{\cdot}q_1)\epsilon_\mu + \{8\mu(p{\cdot}\epsilon n{\cdot}(q_1 - q_2) \\
&- n{\cdot}\epsilon p{\cdot}(q_1 - q_2)) + 84(p{\cdot}n + m^2)\mu\epsilon{\cdot}q_1)K\mu + (8\mu(p{\cdot}\epsilon n{\cdot}(q_1 + K) \\
&- n{\cdot}\epsilon p{\cdot}(q_1 + K)) + 20(p{\cdot}n + m^2)\mu\epsilon{\cdot}(q_1 - K)q_{2\mu} + (8_\mu(n{\cdot}\epsilon p{\cdot}(K + q_2)
\end{aligned}$$

$$-p{\cdot}\epsilon n{\cdot}(K+q_2))+84(p{\cdot}n+m^2)\mu\epsilon{\cdot}K)q_{1\mu}\}\langle\overline{N}_2\gamma_\mu N_1\rangle$$
$$-(p{\cdot}n+m^2)\mu(12K_\mu\epsilon_\nu q_{2\rho}+52K_\mu\epsilon_\nu q_{1\rho}+12\epsilon_\mu q_{2\nu}q_{1\rho})\epsilon_{\mu\nu\rho\alpha}\langle\overline{N}_2\gamma_\alpha\gamma_5 N_1\rangle$$
$$+\{(20(p{\cdot}n+m^2)q_2{\cdot}q_1+8(p{\cdot}q_2 n{\cdot}q_1-p{\cdot}q_1 n{\cdot}q_2-m\mu(p+n){\cdot}(q_2-q_1))$$
$$+84(p{\cdot}n+m^2)\mu^2)K_\mu\epsilon_\nu+(-(p{\cdot}n+m^2)(84\mu^2+20K{\cdot}q_2)$$
$$+8(p{\cdot}Kn{\cdot}q_2-p{\cdot}q_2 n{\cdot}K)-8m\mu(p+n){\cdot}(K+q_2)-16m\mu(n{\cdot}K+m\mu))\epsilon_\mu q_{1\nu}$$
$$+(20(p{\cdot}n+m^2)(\mu^2+K{\cdot}q_1)+8(p{\cdot}Kn{\cdot}q_1+p{\cdot}q_1 n{\cdot}K-m\mu(n+p){\cdot}(K+q_1)))\epsilon_\mu q_{2\nu}$$
$$+(-20(p{\cdot}n+m^2)\epsilon{\cdot}q_1+8(p{\cdot}q_1 n{\cdot}\epsilon-p{\cdot}\epsilon n{\cdot}q_1+m\mu(p+n){\cdot}\epsilon))K_u q_{2\nu}$$
$$+(8(p{\cdot}\epsilon n{\cdot}q_2-p{\cdot}q_2 n{\cdot}\epsilon-m\mu(p+n){\cdot}\epsilon)K_\mu q_{1\nu}$$
$$+(20(p{\cdot}n+m^2)\epsilon{\cdot}K+8(p{\cdot}Kn{\cdot}\epsilon-p{\cdot}\epsilon n{\cdot}K-m\mu(p+n){\cdot}\epsilon))q_{2\mu}q_{1\nu}\}\langle\overline{N}_2 i\sigma_{\mu\nu}N_1\rangle.$$

N_0 is a normalization constant. Note the complexity due to the presence of ϵ_μ, the K^{*-} polarization vector, (cf Adjei *et al* 1971, 1972 Appendix). Also we get more than one Petersson–Törnqvist like kinematic factor with the difference that ϵ_μ always enters instead of one of the external momenta (not surprising since we have used the fact that ϵ_μ is perpendicular to the momentum of K^{*-}).

Applying the same procedure to Kb we are led to taking the trace over four different U(2,2) terms, the sum of which is

$$m^2\mu^3 K_b = N_0 2(m^2+p{\cdot}n)(15K{\cdot}q_2 q_1{\cdot}\epsilon+7\mu^2 q_1{\cdot}\epsilon-22q_2{\cdot}q_1 K{\cdot}\epsilon)+(2m^2+14p{\cdot}n)\mu^2 K{\cdot}\epsilon$$
$$+4m\mu(-2n{\cdot}q_1-p{\cdot}q_1+2p{\cdot}q_2)K{\cdot}\epsilon+(p{\cdot}q_2-p{\cdot}K+n{\cdot}q_2-m{\cdot}K)q_1{\cdot}\epsilon$$
$$-(2p+n){\cdot}\epsilon K{\cdot}q_2+(p+n){\cdot}\epsilon K{\cdot}q_1+(p+n){\cdot}\epsilon q_2{\cdot}q_1-8p{\cdot}q_2 n{\cdot}q_1 K{\cdot}\epsilon$$
$$+4(p{\cdot}Kn{\cdot}q_2-p{\cdot}q_2 n{\cdot}K)q_1{\cdot}\epsilon+4(6p{\cdot}\epsilon n{\cdot}q_1-p{\cdot}q_1 n{\cdot}\epsilon)K{\cdot}q_2$$
$$+4(p{\cdot}q_2 n{\cdot}\epsilon-p{\cdot}\epsilon n{\cdot}q_2)K{\cdot}q_1+4(p{\cdot}\epsilon n{\cdot}K-p{\cdot}Kn{\cdot}\epsilon)q_2{\cdot}q_1+12m\mu^3 n{\cdot}\epsilon$$
$$+4\mu^2(p{\cdot}\epsilon n{\cdot}(2q_1-K-q_2)+p{\cdot}(K+q_2)n{\cdot}\epsilon-4p{\cdot}nK{\cdot}\epsilon)\}\langle\overline{N}_2 N_1\rangle$$
$$+\{4\mu^2(q_{2\mu}\epsilon_\nu-\epsilon_\mu K_\nu)p_\rho n_\lambda-4m\mu q_{2\mu}\epsilon_\nu K_\rho(p_\lambda+n_\lambda)\}\epsilon_{\mu\nu\rho\lambda}\langle\overline{N}_2\gamma_5 N_1\rangle$$
$$+\{(4\mu(p{\cdot}Kn{\cdot}(2q_1+q_2)+p{\cdot}q_1 n{\cdot}(q_2+K)+p{\cdot}q_2 n{\cdot}(2q_1-K)$$
$$-3m\mu p{\cdot}(K+q_2))+2(m^2+p{\cdot}n)\mu\ldots+2(m^2+p{\cdot}n)\mu(K{\cdot}q_1+3K{\cdot}q_2$$
$$+7q_1{\cdot}q_2+4\mu^2)\epsilon_\mu+\ldots(K{\cdot}q_1+3K{\cdot}q_2+7q_1{\cdot}q_2+4\mu^2)\epsilon_\mu$$
$$+(4\mu(3m\mu p{\cdot}\epsilon-2p{\cdot}\epsilon n{\cdot}q_1-p{\cdot}q_1 n{\cdot}\epsilon)$$
$$+14\mu(m^2+p{\cdot}n)q_1{\cdot}\epsilon)K\mu+(4\mu(3m\mu p{\cdot}\epsilon+p{\cdot}\epsilon n{\cdot}(K{\cdot}2q_1)$$
$$-p{\cdot}q_1 n{\cdot}\epsilon)+(m^2+p{\cdot}n)\mu(2q_1{\cdot}\epsilon-10K{\cdot}\epsilon))q_{2\mu}+(4\mu(p{\cdot}(q_2+K)n{\cdot}\epsilon$$
$$-p{\cdot}\epsilon n{\cdot}(q_2+K)+14\mu(m^2+p{\cdot}n)K{\cdot}\epsilon)q_{1\mu}\}\langle\overline{N}_2\gamma_\mu N_1\rangle$$
$$+\{(-K_\mu\epsilon_\nu q_{2\rho}-K_\mu\epsilon_\nu q_{1\rho}-\epsilon_\mu q_{2\nu}q_{1\rho})\epsilon_{\mu\nu\rho\alpha}+(4mq_{2\mu}\epsilon_\nu K_\rho(p_\lambda+n_\lambda)$$
$$-4\mu(q_{2\mu}\epsilon_\nu-\epsilon_\mu K_\nu)p_\rho n_\lambda)\epsilon_{\mu\nu\rho\lambda}q_{1\alpha}\}\langle\overline{N}_2\gamma_\alpha\gamma_5 N_1\rangle$$
$$+\{(4m\mu(p{\cdot}q_1+2p{\cdot}q_2-2nq_1)+6(m^2-p{\cdot}n)\mu^2-8p{\cdot}q_2 n{\cdot}q_1$$
$$+6(m^2+p{\cdot}n)q_2{\cdot}q_1)K_\mu\epsilon_\nu+(4m\mu(2n{\cdot}q_1+2p{\cdot}K-n{\cdot}K-p{\cdot}q_1)$$
$$-4p{\cdot}q_1 n{\cdot}K-6m^2\mu^2-2\mu^2 n{\cdot}p+6(m^2+p{\cdot}n)K{\cdot}q_1)\epsilon_\mu q_2$$

$$+(4m\mu(p+n)\cdot(K-q_2)+4(p\cdot q_2 n\cdot K-p\cdot Kn\cdot q_2)-6(m^2+p\cdot n)(K\cdot q_2+\mu^2)\epsilon_\mu q_{1\nu}$$
$$+(4m\mu(n\cdot\epsilon-2p\cdot\epsilon)+4(p\cdot q_1 n\cdot\epsilon-2p\cdot\epsilon n\cdot q_1)-6(m^2+p\cdot n)q_1\cdot\epsilon)K_\mu q_{2\nu}$$
$$+(-4m\mu(p+n)\cdot\epsilon+4(p\cdot\epsilon n\cdot q_2-p\cdot q_2 n\cdot\epsilon))K_\mu q_{1\nu}+(-4m\mu(p+n)\cdot\epsilon$$
$$+4(p\cdot Kn\cdot\epsilon-p\cdot\epsilon n\cdot K)+6(m^2+p\cdot n)K\cdot\epsilon)q_{2\mu}q_{1\nu}\}\langle\overline{N}_2 i\sigma\mu\nu N_1\rangle.$$

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